

Preheating and Supergravity

Lev Kofman

*Canadian Institute for Theoretical Astrophysics
University of Toronto, M5S 3H8 Canada*

Abstract. In this talk ¹ recent developments of the theory of preheating after inflation are briefly reviewed. In inflationary cosmology, the particles constituting the Universe are created after inflation due to their interaction with moving inflaton field(s) in the process of reheating. In inflationary models motivated by supergravity, both bosons and fermions are created. In the bosonic sector, the leading channel of particle production is the non-perturbative regime of parametric resonance dominated by those bosons which are created exponentially fast with the largest characteristic exponent. In the fermionic sector, the leading channel corresponds to the regime of parametric excitation of fermions, which respects Pauli blocking but differs significantly from the perturbative expectation. In supergravity we also have to consider production of gravitinos and moduli fields, which are cosmologically dangerous relics. We discuss the derivation of the gravitino equations in curved space-time with moving background scalars. We describe recent results on the production of gravitinos from preheating, which may put strong constraints on the inflationary models.

Preheating after Inflation

According to the inflationary scenario, the Universe initially expands quasi-exponentially in a vacuum-like state without entropy or particles. At the stage of inflation, all energy is contained in a classical slowly moving inflaton field ϕ . The fundamental Lagrangian $\mathcal{L}(\phi, \chi, \psi, A_i, h_{ik}, \dots)$ contains the inflaton part with the potential $V(\phi)$ and other fields which give subdominant contributions to gravity. The Friedmann equation for the scale factor $a(t)$ and the Klein-Gordon equation for $\phi(t)$ determine the evolution of the background fields. In most of the models, soon after the end of inflation, an almost homogeneous inflaton field $\phi(t)$ coherently oscillates with a very large amplitude of the order of the Planck mass around the minimum of its potential. This scalar field can be considered as a coherent superposition of inflatons with zero momenta. The amplitude of oscillations gradually

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decreases not only because of the expansion of the universe, but also because energy is transferred to particles created by the oscillating field. At this stage we shall recall the rest of the fundamental Lagrangian which includes all the fields interacting with inflaton. These interactions lead to the creation of many ultra-relativistic particles from the inflaton. Gradually, the inflaton field decays and transfers all of its energy non-adiabatically to the created particles. In this scenario all the matter constituting the universe is created from this process of reheating. If the creation of particles is sufficiently slow, the particles would simultaneously interact with each other and come to a state of thermal equilibrium at the reheating temperature T_r . This gradual reheating can be treated with the perturbative theory of particle creation and thermalization. However, typically particle production from coherently oscillating inflatons occurs not in the perturbative regime but in the non-perturbative regime of parametric excitation. Indeed, let us consider a simple toy model of chaotic inflation with the quadratic potential $V(\phi) = \frac{1}{2}m_\phi\phi^2$ and $\mathcal{L}_{int} = -\frac{1}{2}g^2\phi^2\chi^2$ describing the interaction between the inflatons and other massless Bose particles χ . The quantum scalar field $\hat{\chi}$ in a flat FRW background has the eigenfunctions $\chi_k(t)e^{-i\mathbf{k}\cdot\mathbf{x}}$ with comoving momentum \mathbf{k} . The temporal part of the eigenfunction obeys the equation

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{\mathbf{k}^2}{a^2} - \xi R + g^2\phi^2\right)\chi_k = 0 \quad (1)$$

with vacuum-like initial conditions: $\chi_k \simeq \frac{e^{-ikt}}{\sqrt{2k}}$ in the far past. The coupling to the curvature ξR will not be important in the presence of the interaction (but would lead to gravitational preheating in the absence of the interaction). In this model, the inflaton field $\phi(t)$ coherently oscillates as $\phi(t) \approx \Phi(t) \sin(m_\phi t)$, with the amplitude $\Phi(t) = \frac{M_p}{\sqrt{3\pi}} \cdot \frac{1}{m_\phi t}$ decreasing as the universe expands. The smallness of g^2 alone does not necessarily lead to the perturbative excitation of χ_k modes. To check whether the interaction term $g^2\phi^2$ in eq. (1) is perturbative or not, it is convenient to use a new time variable $z = mt$ and the essential dimensionless coupling parameter $q = \frac{g^2\Phi^2}{m^2}$. Scalar metric fluctuations in this model are compatible with cosmology if the inflaton mass is $m \simeq 10^{-6}M_p$; therefore, it is expected that $q \simeq 10^{10}g^2 \gg 1$ for not negligibly small g^2 . In fact, a consistent setting for the problem of χ -particle creation from the ϕ -inflaton requires $q \gg 1$ even without additional assumptions about g^2 [2]. Indeed, it is known that if we have two scalars ϕ and χ , then the latest stage of inflation will be driven by the lightest scalar. The square of the effective mass of the χ -field includes a term $g^2\phi^2$. Inflation is driven by the ϕ -field if its square mass m^2 is smaller than $g^2\phi^2$. This leads to the condition $q \gg 1$, i.e. to the creation of χ -particles in the resonance regime.

Supergravity and the Early Universe

To make the next step beyond toy models of particles interactions with the inflaton, we have to choose the fundamental Lagrangian $\mathcal{L}(\phi, \chi, \psi, A_i, h_{ik}, \dots)$. We

may expect that the low-energy physics of the early universe will be described by the general four-dimensional $N = 1$ supergravity-Yang-Mills-matter theory [1]. A rather lengthy $N = 1$ phenomenological supergravity Lagrangian ² begins with the terms

$$\begin{aligned}
e^{-1}\mathcal{L} = & -\frac{1}{2}M_P^2 R - \hat{\partial}_\mu \Phi^i \hat{\partial}^\mu \Phi_i + e^K \left(\mathcal{D}^i W \mathcal{D}_i W - 3 \frac{WW^*}{M_P^2} \right) \\
& - \bar{\chi}_j \mathcal{D} \chi^i - \bar{\chi}^i \mathcal{D} \chi_j - \left(e^{K/2} \mathcal{D}^i \mathcal{D}^j W \bar{\chi}_i \chi_j + h.c. \right) \\
& - \frac{1}{2} \bar{\psi}_\mu R^\mu + \left(\frac{1}{2} e^{K/2} W \bar{\psi}_{\mu R} \gamma^{\mu\nu} \psi_{\nu R} + \bar{\psi}_{\mu L} \hat{\partial} \Phi^i \gamma^\mu \chi_i + \bar{\psi}_R \chi_i e^{K/2} \mathcal{D}^i W + h.c. \right) \\
& + \dots
\end{aligned} \tag{2}$$

A particular choice of the form of the Lagrangian is motivated and notations are given in [3]. In Eq. (2) we choose the minimal Kähler potential $K = \frac{\Phi_i \Phi^i}{M_P^2}$, where Φ^i is the complex conjugate of Φ_i . The last term of the 1st line is the scalar potential $V(\Phi_i)$. The equations of motion based on the first line should describe inflation, which is a challenging problem by itself. For simplicity we will take a prototype model of the superpotential $W = \sqrt{\lambda} \Phi^3/3$, which for $|\Phi| \ll M_P$ leads to the effective potential $\lambda \Phi^4$. We will illustrate some effects of preheating in supergravity with this model.

Preheating of Bosons

Let us briefly recall the basics of the bosonic preheating. Consider the creation of χ -particles due to the $g^2 \chi^2 \phi^2$ interaction. To get a feeling for the way in which χ -particles are created from inflaton oscillations, we have to understand the character of solutions of eq. (1) for the mode functions $\chi_k(t)$. For preheating of bosons we need the first line of the Lagrangian (2). Consider the model with the inflaton potential $V(\phi) = \frac{1}{4} \lambda \phi^4$ in an expanding universe. The problem of particle creation in this theory can be reduced to a similar problem in Minkowski space-time. This can be realized with the conformal transformation of the scalar field $\phi \rightarrow \varphi = a\phi$ and with the conformal time variable $\tau = \sqrt{\lambda} \tilde{\varphi} \int \frac{dt}{a(t)}$. Therefore, $\frac{1}{4} \lambda \phi^4$ theory is sometimes dubbed the conformal theory. In conformal variables, the Klein-Gordon equation for $\varphi(\tau)$ is reduced to an equation in flat space-time. Its solution is $\varphi(\tau) \approx \tilde{\varphi} f(\tau)$, where the amplitude of the oscillations $\tilde{\varphi}$ is constant until the backreaction of created particles is taken into account. The time-dependence of the oscillations in this theory is not sinusoidal, but given by an elliptic function $f(\tau) = cn\left(\tau, \frac{1}{\sqrt{2}}\right)$. Eq. (1) for quantum fluctuations χ_k can be simplified in this theory. Using a conformal transformation of the mode function $X_k(t) = a(t)\chi_k(t)$ in Eq. (1) we obtain

²) When I show a viewgraph with the full supergravity Lagrangian at the cosmology conferences, somehow people start to laugh hysterically.

$$X_k'' + (\kappa^2 + qf^2(\tau))X_k = 0, \quad (3)$$

where $\kappa^2 = \frac{k^2}{\lambda\bar{\varphi}^2}$, and $q = \frac{g^2}{\lambda}$. The equation for fluctuations does not depend on the expansion of the universe and is completely reduced to the similar problem in Minkowski space-time. This is a special feature of the conformal theory $\frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$. The mode equation (3) belongs to the class of Lamé equations. The combination of parameters $q = g^2/\lambda$ ultimately defines the structure of the parametric resonance in this theory. This means that the condition of a broad parametric resonance does not require a large initial amplitude of the inflaton field, as for the quadratic potential. The strength of the resonance depends non-monotonically on the value of the q parameter. The stability/instability chart of the Lamé equation (3) in the variables $(\kappa^2, \frac{q^2}{\lambda})$ was constructed in [6]. To see how the general theory works, let us consider parametric resonance of inflaton fluctuations ϕ_k due to the self-interaction of inflaton field in $\lambda\phi^4$ theory. Using conformal transformation of the mode functions $\varphi_k(\tau) = a(t)\phi_k(t)$, the equation for φ_k can be reduced to the general equation (3) with the particular value $q = 3$. The equation for fluctuations X_k when $q = 3$ can be solved analytically [6]. The resonance in the “inflaton” direction ϕ is weak; the maximal value of the characteristic exponent of the fluctuations $\phi_k \propto e^{\mu_1\tau}$ is $\mu \approx 0.036$. Let us however consider a supersymmetric version of the conformal theory. In the first line of the Lagrangian (2) we will put the superpotential $W = \sqrt{\lambda}\Phi^3/3$, which at $\phi \ll M_P$ gives us a scalar potential with the conformal properties $V(\Phi) = \frac{1}{4}(\Phi^2)^2$. In supersymmetric theories, all scalar are complex. The scalar field Φ will have two components $(\phi, \bar{\phi})$. Let us assume that inflaton direction corresponds to the real component ϕ , $\text{Re } \Phi = \frac{\phi}{\sqrt{2}}$, and initially $\bar{\phi} = 0$, $\text{Im } \Phi = 0$. The equation for the mode function of fluctuations in the direction $\bar{\phi}$ can be obtained with the conformal transformation $\bar{\varphi}_k(\tau) = a(t)\bar{\phi}_k(t)$ and can be reduced reduces to the general equation (3) with the parameter $q = 1$. Again, in this case the problem can be solved analytically [6]. The factor $q = 1$ instead of 3 in eq. (3) makes a big difference, which manifests the subtlety of the parametric resonance. The resonance in the direction $\bar{\phi}$ is much stronger and broader than the resonance in the inflaton direction, $\bar{\phi}_k \propto e^{\bar{\mu}\tau}$, with $\bar{\mu} \approx 0.147$. Thus, the supergravity generalization of bosonic preheating in the conformal theory makes a big difference in the bosonic preheating due to the self-interaction. Note that the character and strength of the parametric resonance for the general equation (3) depends on the shape of the effective inflaton potential $V(\phi)$. Moreover, the investigation of the resonance in an expanding universe typically cannot be reduced to the study of the regular stability/instability chart. If theory is not conformal, (say due to the mass term $m^2\phi^2$ in $V(\phi)$) the parameter q in an expanding universe is time-dependent. For the broad resonance case $q \gg 1$ this parameter can jump over a number of instability bands within a single oscillation of the inflaton field, and the concept of stability/instability bands is inapplicable here. Parametric resonance in this case is a stochastic process [5].

Parametric Excitations of Fermions

A simple model for the inflaton's interaction with Fermi particles χ is a Yukawa term $h\bar{\chi}\phi\chi$. For instance, consider second line of eq. (2) and ignore mixing between χ and ψ_μ (which corresponds to the rigid SUSY limit). For our toy model with $W = \frac{1}{3}\sqrt{\lambda}\Phi^3$ and $\phi \ll M_P$, the “mass” term of the chiral fermion χ is equal to $\sqrt{2\lambda}\phi$, which corresponds to the inflaton-fermion interaction $\sqrt{2\lambda}\bar{\chi}\phi\chi$. For fermions, the Pauli exclusion principle prohibits the occupation number from exceeding 1. For this reason, it has been silently assumed that fermions are created in the three-legs perturbative process $\phi \rightarrow \bar{\chi}\chi$ where individual inflatons decay independently into pairs of ψ -particles. Let us, however, consider the Dirac equation for a massless quantum Fermi field $\chi(t, \vec{x})$:

$$[\gamma^\mu \nabla_\mu + h\phi(t)]\chi = 0, \quad (4)$$

where ∇_μ is the derivative with the spin connection. We are using the representation of gamma matrices where $\gamma_0 = \text{diag}(i, i, -i, -i)$. Here, similar to the bosonic case, the inflatons producing fermions also act not as individual particles but as a coherently oscillating field $\phi(t)$. Let us consider more general model $\frac{1}{4}\lambda\phi^4 + h\bar{\psi}\phi\psi$. This is a conformal theory in the sense that the problem of fermion production by the inflaton ϕ in an expanding universe can be reduced to equations in Minkowski space-time. Indeed, let us perform a conformal transformation of the involved fields, $\varphi \equiv a\phi$ and $\Psi \equiv a^{3/2}\chi$, and use a conformal time variable, τ as in the previous section. The equation for the eigenfunctions of the quantum fluctuations in this theory can be reduced to a second-order equation for an auxillary field $X(\tau, \vec{x})$, so that $\Psi = [\gamma^\mu \nabla_\mu + h\varphi]X$. The eigenmodes of the auxillary field have the form $X_k(\tau)e^{+ik\cdot\vec{x}}R_r$, where the R_r are eigenvectors of the Dirac matrix γ^0 with eigenvalue $+1$. The temporal part of the eigenmode obeys an oscillator-like equation with a complex frequency which depends periodically on time

$$X_k'' + (\kappa^2 + qf^2 - i\sqrt{q}f')X_k = 0. \quad (5)$$

The dimensional comoving momentum k enters the equation in the combination $\frac{k^2}{\lambda\tilde{\varphi}^2} \equiv \kappa^2$; therefore, the natural units of momentum are $\sqrt{\lambda}\tilde{\varphi}$. The background oscillations enter in the form $f(\tau)$ of the previous section, having unit amplitude. The imaginary part of the frequency in Eq. (5) guarantees the Pauli blocking for the occupation number n_k . The results for n_k can be formulated as follows [4]. Even though the Yukawa interaction contains a small factor h , one cannot use the perturbation expansion in h . This is because the frequency of the background field oscillations is proportional to another small parameter $\sqrt{\lambda}$. The combination of the coupling parameters $\frac{h^2}{\lambda} \equiv q$ ultimately determines the strength of the effect. The growth of fermionic modes occurs in the non-perturbative regime of parametric excitation. The modes get fully excited with occupation numbers $n_k \simeq 1$ within tens of oscillations of the field ϕ , and the width of the parametric excitation of fermions

in momentum space is about $q^{1/4}\sqrt{\lambda}\phi_0$. For instance, in the case of $h = \sqrt{2\lambda}$, $q = 2$, the modes will be excited in about ten oscillations, and the width will be about $\sqrt{\lambda}\phi_0$.

Equations for the Gravitino

The rest of my talk is based on the recent project [3]³. Let us now consider the third line in the supergravity Lagrangian (2), which describes the gravitino field ψ_μ . In a general background metric and in the presence of complex scalar fields with non-vanishing VEV's, the equation for the gravitino has on the left hand side the kinetic part $R^\mu \equiv \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma$, and a rather lengthy right hand side. We will use the long derivative \mathcal{D}_μ with the spin connection and Christoffel symbols, for which $\mathcal{D}_\mu\gamma_\nu = 0$. Apart of varying gravitino mass m , the right hand side contains a chiral connection and various mixing terms like those in the 3rd line of (2). For a self-consistent setting of the problem, the gravitino equation should be supplemented by the equations for the fields mixing with gravitino, χ_i from (2), as well as by the equations determining the gravitational background and the evolution of the scalar fields. Let us make some simplifications. We consider the supergravity multiplet and a single chiral multiplet containing a complex scalar field Φ with a single chiral fermion χ . This is a simple non-trivial extension which allows us to study the gravitino with a non-trivial FRW cosmological metric supported by the scalar field. A nice feature of this model is that the chiral fermion χ can be gauged to zero so that the mixing between ψ_μ and χ in (2) is absent. We also can choose the non-vanishing VEV of the scalar field (inflaton) in the real direction, as in the previous sections. First we will derive the equation for a spin 3/2 field in a curved background metric with non-vanishing VEVs for the scalar fields.

1 Spin 3/2 Field Equations in External gravitational and Scalar Fields

From (2) we can obtain the equation for the gravitino field (in case of single chiral multiplet χ and vanishing $\text{Im } \Phi$) $R^\mu = m(\tau)\gamma^{\mu\nu}\psi_\nu$, where $\gamma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ and gravitino mass $m = m(\phi(\tau))$ is given by $m = e^{K/2} \frac{W}{M_P^2}$. This equation can be transformed into the form

$$\mathcal{D}\psi_\mu + m\psi_\mu = \left(\mathcal{D}_\mu - \frac{m}{2}\gamma_\mu\right)\gamma^\nu\psi_\nu. \quad (6)$$

The gravitino equation (6) is a curved spacetime generalization of the familiar gravitino equation $(\not{\partial} + m_0)\psi_\mu = 0$ in a flat metric, where m_0 is a constant gravitino mass. The generalization of the first constraint equation $\partial^\mu\psi_\mu = 0$ can be obtained from the equality $\gamma_\mu R^\mu = 2\gamma^{\mu\nu}\mathcal{D}_\mu\psi_\nu$ and reads

³) All credit for the correct results below go to [3]. All incorrect deviations is my fault.

$$\mathcal{D}^\mu \psi_\mu - \mathcal{D} \gamma^\mu \psi_\mu + \frac{3}{2} m \gamma^\mu \psi_\mu = 0. \quad (7)$$

The generalization of the second constraint equation $\gamma^\mu \psi_\mu = 0$ can be obtained from the equality $\mathcal{D}_\mu R^\mu = -\frac{1}{2} G_{\nu\rho} \gamma^\nu \psi^\rho$ (dropping the torsion term) and is

$$\frac{3}{2} m^2 \gamma^\mu \psi_\mu + (\gamma^\mu \partial_\mu m) \gamma^\nu \psi_\nu - (\partial_\mu m) \psi^\mu = -\frac{1}{2} G_{\mu\nu} \gamma^\mu \psi^\nu. \quad (8)$$

In our case $m = m(\tau)$ and from (8) one can find an algebraic relation between $\gamma^0 \psi_0$ and $\gamma^i \psi_i$:

$$\gamma^0 \psi_0 = \hat{A} \gamma^i \psi_i. \quad (9)$$

Here \hat{A} is a matrix which will play a crucial role in our description of the interaction of the gravitino with the varying background fields. If ρ and p are the background energy-density and pressure, we have $G_0^0 = M_P^{-2} \rho$, $G_k^i = -M_P^{-2} p \delta_k^i$, and one can represent the matrix \hat{A} as follows:

$$\hat{A} = \frac{p - 3m^2 M_P^2}{\rho + 3m^2 M_P^2} + \gamma_0 \frac{2m' a^{-1} M_P^2}{\rho + 3m^2 M_P^2} = A_1 + \gamma_0 A_2. \quad (10)$$

We shall solve the equation (6) using the constraint equations in the form (8) and (9). We use a plane-wave ansatz $\psi_\mu \sim e^{i\mathbf{k}\cdot\mathbf{x}}$ for the space-dependent part. Then ψ_i can be decomposed into its transverse part ψ_i^T , and to the longitudinal part ψ_i^L which is defined by the trace $\gamma^i \psi_i$. Two degrees of freedom of ψ_μ are associated with the transverse part ψ_i^T , which correspond to helicity $\pm 3/2$ and two degrees of freedom are associated with $\gamma^i \psi_i$ (or ψ_0) which correspond to helicity $\pm 1/2$. For the helicity $\pm 3/2$ states we have to derive the equation for ψ_i^T . We apply decomposition $\psi = \psi_i^T + \psi_i^L$ to the master equation (6) for $\mu = i$ and obtain

$$\left(\gamma^\mu \partial_\mu + \frac{a'}{2a} \gamma^0 + ma \right) \psi_i^T = 0. \quad (11)$$

The transformation $\psi_i^T = a^{-1/2} \Psi_i^T$ reduces the equation for the transverse part to the free Dirac equation with a time-varying mass term ma , c.f. eq. (4). In the previous section we explained how to treat this type of equation. The essential part of Ψ_i^T is given by the time-dependent part of the eigenmode of the transversal component $X_T(\eta)$, which obeys second-order equation (c.f. (5)):

$$X_T'' + (k^2 + (ma)^2 - i(ma)') X_T = 0. \quad (12)$$

The corresponding equation for gravitino with helicity $1/2$ is more complicated. We have to find $k^i \psi_i$ and $\gamma^i \psi_i$. The equation for the components $k^i \psi_i$ can be obtained from the constraint equation (7). The equation for $\gamma^j \psi_j$ can be derived from (6). Using ψ_0 from (9), we get an equation for $\gamma^i \psi_i$

$$(\partial_\eta + \hat{B} - i\mathbf{k} \cdot \gamma \gamma_0 \hat{A}) \gamma^i \psi_i = 0, \quad (13)$$

where $\hat{B} = -\frac{3a'}{2a}\hat{A} - \frac{ma}{2}\gamma_0(1 + 3\hat{A})$. The time-dependent factor of the spinor $\gamma^i \psi_i$, which we denote as $f_k(\tau)$, obeys a second-order differential equation. By the substitution $f_k(\tau) = E(\eta)X_L(\eta)$, with $E = (-A^*)^{1/2} \exp(-\int^\eta d\eta \operatorname{Re} B)$, the equation for the function $f_k(\tau)$ is reduced to the final oscillator-like equation for the time-dependent mode function $X_L(\eta)$:

$$X_L'' + (|A|^2 k^2 + \Omega_L^2 - i\Omega_L') X_L = 0. \quad (14)$$

Here $a^{-1}\Omega_L = \frac{i}{2}\partial_\tau \ln A^* + \frac{3a'}{2a}A_2 + \frac{1}{2}ma(-1 + 3A_1)$. For an arbitrary background FRW metric $a(\tau)$ and background scalar field $\phi(\tau)$, equation (14) may lead to ill-defined physics. For instance, if $|A| > 1$, it describes noncasual propagation of spin 3/2 particles. In the context of preheating, the background fields $\phi(\tau)$ and $a(\tau)$ are oscillating. Naively, one could expect that $|A|$ is oscillating too. In this case one would reach a pathological conclusion that the strongest parametric excitation of spin 3/2 eigenmodes will be with the highest momenta k . Noncausality and other defects of spin 3/2 fields interacting with an external electromagnetic field are well known [7]. It is interesting to compare the equations for a spin 3/2 field in an external electromagnetic field and in external gravitational/scalar fields. Then we will show how the apparent defects of (14) are resolved in supergravity.

2 Spin 3/2 Field Equations in an External Electromagnetic Field

The equations for a charged spin 3/2 field interacting with an external electromagnetic field A_μ with field strength tensor $F_{\mu\nu}$ in flat space-time were derived in [7]. After simple manipulations, they can be re-written in the form of the equation of motion

$$\hat{\mathcal{D}}\psi_\mu + m_0\psi_\mu = \left(\hat{\mathcal{D}}_\mu - \frac{m_0}{2}\gamma_\mu\right) \gamma^\nu \psi_\nu, \quad (15)$$

and two constraint equations

$$\hat{\mathcal{D}}^\mu \psi_\mu - \hat{\mathcal{D}}\gamma^\mu \psi_\mu + \frac{3}{2}m_0\gamma^\mu \psi_\mu = 0, \quad (16)$$

$$\frac{3}{2}m_0^2\gamma^\mu \psi_\mu = -\frac{1}{2}\tilde{F}^{\mu\nu}\gamma_\mu \psi_\nu, \quad (17)$$

where $\tilde{F}^{\mu\nu} = e\gamma^5\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. Here $\hat{\mathcal{D}}_\mu \equiv \partial_\mu - ieA_\mu$. The system of equations (15), (16), (17) for flat case with EM background is similar to the equations (6), (7), (8) for cosmological problem up to definition of derivatives $\hat{\mathcal{D}}_\mu$ instead of \mathcal{D}_μ , a constant mass m_0 instead of the altering mass $m(\tau)$, and $\tilde{F}^{\mu\nu}$ instead of $G^{\mu\nu}$. It is interesting that a spin 3/2 field interacting with external fields can be described in the unified way. In [7] it was shown that for some configurations of the electromagnetic field, the propagation of spin 3/2 field violates causality.

3 Self-consistent Gravitino Problem in Supergravity

Contrary to the inconsistent setting of the problem of a spin 3/2 field in an arbitrary EM or gravitational field, the spin 3/2 gravitino field in supergravity should be consistent. In this section we will show that the gravitino equations in an expanding universe with moving scalars are consistent. We will use a model with a single chiral multiplet. Let us concentrate on the matrix \hat{A} given by (10). In the models where the energy-momentum tensor is determined by the energy of a classical scalar field and Φ depends only on time we have $\rho = |\dot{\Phi}|^2 + V$, $p = |\dot{\Phi}|^2 - V$. The scalar potential is $V(\Phi) = e^K |\mathcal{D}W|^2 - 3m^2 M_P^2$. Also, we have $m' = ae^{K/2} \mathcal{D}W \dot{\Phi} / M_P^2$. Therefore, the matrix \hat{A} can be rewritten in terms of $\dot{\Phi}$ and $e^{K/2} \mathcal{D}W$ only

$$\hat{A} = \frac{|\dot{\Phi}|^2 - |e^{K/2} \mathcal{D}W|^2}{|\dot{\Phi}|^2 + |e^{K/2} \mathcal{D}W|^2} + \gamma_0 \frac{2\dot{\Phi} e^{K/2} \mathcal{D}W}{|\dot{\Phi}|^2 + |e^{K/2} \mathcal{D}W|^2}. \quad (18)$$

From this form of \hat{A} it follows that

$$|A|^2 \equiv A_1^2 + A_2^2 = 1 \quad (19)$$

for an arbitrary superpotential W . Thus A can be represented as $A = -\exp\left(2i \int_{-\infty}^t dt \mu(\eta)\right)$. Using the Einstein equations, one obtains $\mu = \mathcal{D}\mathcal{D}W + \Delta$, where the correction $\Delta = \mathcal{O}(M_P^{-1})$ is given in [3]. The expression for μ becomes much simpler and its interpretation is more transparent if the amplitude of oscillations of the field Φ is much smaller than M_P . In the limit $\Phi/M_P \rightarrow 0$ one has $\mu = \partial_\Phi \partial_\Phi W$. This coincides with the mass of both fields of the chiral multiplet (the scalar field and spin 1/2 fermion) in rigid supersymmetry. When supersymmetry is spontaneously broken, the chiral fermion, goldstino, is ‘eaten’ by the gravitino which becomes massive and acquires helicity $\pm 1/2$ states in addition to the helicity $\pm 3/2$ states of the massless gravitino. With this form of \hat{A} the gravitino equation (14) becomes

$$X_L'' + \left(k^2 + \Omega_L^2 - i\Omega_L'\right) X_L = 0. \quad (20)$$

with $a^{-1}\Omega_L = \mu - \frac{3}{2}H \sin 2\int \mu dt - \frac{1}{2}m(1 + 3\cos 2\int \mu dt)$. Equation (20) is consistent and describes the creation of gravitinos from preheating. The solution of the consistency problem in the equation (20) is that the symmetries imprinted to the Supergravity provide $|A| = 1$. But it does not mean that A is a constant. The matrix \hat{A} does not become constant even in the limit $M_P \rightarrow \infty$. The phase of \hat{A} rotates when the background scalar field oscillates. The amplitude and sign of A change two times within each oscillation. Consequently, the relation between $\gamma^0 \psi_0$ and $\gamma^i \psi_i$ also oscillates during the field oscillations. This means that the gravitino with helicity 1/2 (which is related to ψ_0) remains coupled to the changing background even in the limit $M_P \rightarrow \infty$. In a sense, the gravitino with helicity 1/2

remembers its goldstino nature. This is the main reason why gravitino production in this background in general is not suppressed by the gravitational coupling. The main dynamical quantity which is responsible for the gravitino production in this scenario will not be the small changing gravitino mass $m(t)$, but the mass of the chiral multiplet μ , which is much larger than m .

Problem of Gravitino Over-Production from Preheating

As an example, consider the model with the superpotential $W = \sqrt{\lambda}\Phi^3/3$. The parameter μ for this model is given by $\mu = \sqrt{2\lambda}\phi$. It rapidly changes in the interval between 0 and $\sqrt{2\lambda}\phi_0$. Initially it is of the same order as H and m , but then H and m rapidly decrease as compared to μ , and therefore the oscillations of μ remain the main source of the gravitino production. In this case the production of gravitinos with helicity 1/2 is much more efficient than that of helicity 3/2. The theory of production of gravitinos with helicity 1/2 in this model is similar to the theory of production of spin 1/2 fermions with mass $\sqrt{2\lambda}\phi$ by the coherently oscillating scalar field in the theory $\lambda\phi^4/4$, which we considered above. Growth of helicity 1/2 gravitino modes (20) occurs in the non-perturbative regime of parametric excitation. The modes get fully excited with occupation numbers $n_k \simeq 1$ within about ten oscillations of the field ϕ , and the width of the parametric excitation of fermions in momentum space is about $\sqrt{\lambda}\phi_0$. This result violates the cosmological constraints on the abundance of gravitinos with mass $\sim 10^2$ GeV by 4 orders of magnitude [3]. The most dangerous gravitino over-production (by 14 orders of magnitude) occurs in the class of inflationary models where $V(\phi)$ does not have a minimum and where preheating is gravitational (NO models) [2]. Thus the investigation of the non-thermal gravitino production in the early universe may serve as a useful tool helping us to discriminate among various versions of the cosmological theory.

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